Beyond Existence: Inferences about Mental Processes from Reversed Associations

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One of the aims of cognitive (neuro)psychology is to characterize the nature of, and the relationships between, mental processes underlying human behavior (c.f. Dunn & Kirsner, this issue). In this context, the double dissociation (DD) paradigm has played an important albeit debated role. In our efforts to come up with a clear and positive contribution to this discussion, which sometimes appears to be a muddle of terms and arguments, we turned to the framework of Dunn and Kirsner (1988). Their framework and analysis is a satisfyingly clear, logical elucidation of the possible meanings of "double dissociation." These authors have shown that a data pattern called the reversed association (RA) (a stronger version of double dissociation) is sufficient to logically infer the existence of at least two processes underlying task behavior. However, some open questions remain. Here we tackle the specific question "What can be inferred from a reversed association beyond the mere existence of two processes?" We will show that the answer to this question depends heavily on assumptions about monotonicity of functional relationships.

In the Dunn-Kirsner (DK) framework, the relationship between variables, processes and tasks can be understood as two nested transformations. First, levels of experimental variables, denoted $v_1, v_2, ..., v_n$, are mapped onto levels of process efficiency, $p_1, p_2, ..., p_m$. Second, p_i are mapped onto levels of task performance, $t_1, t_2, ... t_k$. Here v_i denotes the level of the i^{th} variable, p_i denotes the efficiency of the i^{th} process, and t_i denotes the performance on the i^{th} task. Functions f_i , with $p_i = f_i(v_1, v_2, ..., v_n)$, for $i = 1, 2, ..., m$, transform variables onto processes, and functions g_i, with $t_i = g_i(p_1, p_2, ..., p_m)$, $i = 1, ..., k$, transform processes onto task performance levels.

Definition 1: A model is a *single-process model* if for all t_i, we can write $t_i = g_i(p)$. A model is a *two-process model* if for all t_i, we can write $t_i = g_i(p_1, p_2)$.

Definition 2: Two processes, p_1 and p_2, are *functionally dependent* (abbreviated *f.d.*) if $p_1 = h(p_2)$. Otherwise, p_1 and p_2, are said to be *functionally independent* (*f.i.*).

In the DK framework, if two processes are functionally dependent, performance on the two tasks becomes functionally related. This is because $t_i = g_i(p_1, p_2) = g_i(p_1,h(p_2)) = G_i(p_1)$ for $i = 1, 2$, so $t_1 = G_1(G_2^{-1}(t_2))$. Thus, in this sense, a two-process model with f.d. processes can be reduced to a single-process model.

Definition 3: A two-process model is *process pure* if $t_1 = g_1(p_1)$, $t_2 = g_2(p_2)$ and $p_1 \neq p_2$. Otherwise, t_1 and t_2 are said to *share* some process(es), e.g., if $t_1 = g_1(p_1,p_2)$ and $t_2 = g_2(p_2)$.

For ease of presentation, we will discuss our analyses as if a single-process model or two-process model were true; i.e., excluding the possibility of the existence of three or more processes underlying behavior on two tasks.[1] Consequently we distinguish seven models, based on the number of processes (one or two), their process purity and their functional independence; see top

[1] Our analyses can of course be extended to allow for inferences about multi-process models in general.

row in Table 1. We now ask, for each model, whether it is consistent with the data pattern that Dunn and Kirsner (1988) called *reversed association*, as we make increasingly strict assumptions on the functions g_1, g_2 and h (shown in the rows of Table 1 as we go down).

Definition 4. A *reversed association* (RA) is the conjunction of two findings: (1) for two variables, v_1 and v_2, $t_1(v_1) > t_1(v_2)$ and $t_2(v_1) > t_2(v_2)$, i.e., there is a positive association between t_1 and t_2, and (2) for two (possibly but no necessarily other) variables, v_3 and v_4, $t_1(v_3) > t_1(v_4)$ and $t_2(v_3) < t_2(v_4)$, i.e., a negative association between t_1 and t_2.

If a model-assumption combination is inconsistent with RA (i.e., the model necessarily refutes the possibility of observing a RA under those assumptions) then an experimenter can reject the model whenever RA is observed, provided that she is willing to make the respective assumptions. This way we can specify which assumptions allow for stronger (or weaker) inferences about the nature of, and relationship between, processes.

Clearly, all models are consistent with RA if g_1, g_2 and h can be any function (assumption 1 in Table 1) or if we assume only one of g_1 or g_2 is monotonic[2] (assumption 2). Dunn and Kirsner (1988) showed that a RA is logically inconsistent with a single-process model if both g_1 and g_2 are assumed to be monotonic functions (assumption 3). Briefly, their argument is that if $t_1 = g_1(p)$ and $t_2 = g_2(p)$, and both g_1 and g_2 are monotonic functions, then, since $\mathrm{d}t_2/\mathrm{d}t_1 = (\mathrm{d}t_2/\mathrm{d}p)(\mathrm{d}p/\mathrm{d}t_1)$, t_1 will be a monotonic function of t_2. This observation is true whether the monotonicity is increasing for both, decreasing for both, or mixed. Hence, an RA under this assumption 3 is sufficient to reject a single-process model.

Table 1.
Single- and two-process models that are consistent (C) or inconsistent (*) with RA (non-monotonic relationship between t_1 and t_2), under various monotonicity assumptions.

		MODELS						
ASSUMPTIONS about functions		*1. Single process model*	*2. Pure f.i. processes*	*3. Pure f.d. processes*	*4. F.i. processes, one shared*	*5. F.d. processes, one shared*	*6. F.i. processes, both shared*	*7. F.d. processes, both shared*
g_1 and g_2	h	$t_1 = g_1(p)$ $t_2 = g_2(p)$	$t_1 = g_1(p_1)$ $t_2 = g_2(p_2)$ p_1, p_2 f.i.	$t_1 = g_1(p_1)$ $t_2 = g_2(p_2)$ $p_1 = h(p_2)$	$t_1 = g_1(p_1, p_2)$ $t_2 = g_2(p_2)$ p_1, p_2 are f.i.	$t_1 = g_1(p_1, p_2)$ $t_2 = g_2(p_2)$ $p_1 = h(p_2)$	$t_1 = g_1(p_1, p_2)$ $t_2 = g_2(p_1, p_2)$ p_1, p_2 are f.i.	$t_1 = g_1(p_1, p_2)$ $t_2 = g_2(p_1, p_2)$ $p_1 = h(p_2)$
1. any function		**C**	**C**	**C**	**C**	**C**	**C**	**C**
2. one mon.	*any*	**C**	**C**	**C**	**C**	**C**	**C**	**C**
3. both mon.	*any*	*	**C**	**C**	**C**	**C**	**C**	**C**
4. both mon.	*mon.*	*	**C**	*	**C**	**C**	**C**	**C**
5. same mon.	*mon.*	*	**C**	*	**C**	**C**	**C**	**C**
6. same monotonic		*	**C**	*	**C**	*	**C**	*

Note: ***** means that the model implies a monotonic relationship between t_1 and t_2, and hence can be rejected if an RA is observed; **C** means that the model does not necessarily imply a monotonic relationship between t_1 and t_2, and hence cannot be rejected if an RA is observed.

[2] We say a function $g(x, y)$ with two arguments, x and y, is *monotonic*, if it is monotonic in x for all fixed y, and it is monotonic in y for all fixed x. In other words, the partial derivate of g with respect to x and the partial derivative of g with respect to y, never changes sign. Further, we say a function $g(x, y)$ is *monotonic increasing* (or *monotonic decreasing*), if it is monotonic increasing (monotonic decreasing) in both arguments.

An interesting question then becomes: If RA is observed under assumption 3, can all two-process models be true or only some of them? Dunn and Kirsner wrote: "[The multi-process model] can be verified, and a reversed association observed, only when variables, processes, and tasks are … functionally independent of one another" (1988, p. 98). If this claim were true, it would imply that only f.i. processes can lead to reversed associations, and thus that RA implies a two-process model with f.i. processes (i.e., either model 2, 4 or 6). Clearly, models 3, 5, and 7 are consistent with RA under assumption 3 where *h* can be any function. With the following counter-example, we further show that models 5 and 7 are also consistent with RA, even if the functional dependence, *h*, is monotonic (assumption 4).

Counter-example:[3] Suppose we have two tasks that are each monotonically related to two processes; i.e., $t_1 = g_1(p_1, p_2)$ and $t_2 = g_2(p_1, p_2)$, and suppose further that the two processes are f.d., such that $p_2 = h(p_1)$ with *h* a monotonic function (i.e., model 7 under assumption 4). Now $t_1 = g_1(p_1, h(p_1)) = G_1(p_1)$, and $t_2 = g_2(p_1, \mathrm{h}(p_1)) = G_2(p_1)$. To show that an RA can be observed under these conditions, it suffices to show that G_1 or G_2 can be non-monotonic. Without loss of generality let $p_1, p_2 \geq 0$. Let g_1 be the following monotonic increasing function of both p_1 and p_2,

$$g_1(p_1, p_2) = p_1 + p_2$$

and let *h* be the monotonic decreasing function,

$$p_2 = h(p_1) = -(p_1)^2$$

then we have

$$G_1(p_1) = g_1(p_1,h(p_1)) = p_1 - (p_1)^2$$

Since $(p_1)^2 \leq p_1$ for $0 \leq p_1 \leq 1$, but $(p_1)^2 > p_1$ for $p_1 > 1$, we can conclude that G_1 is not a monotonic function of p_1. In other words, $\mathrm{d}G_1/\mathrm{d}p_1 = 1 - 2p_1$ is positive for $0 \leq p_1 < 0.5$ and negative for $p_1 > 0.5$. QED.

In sum, the counter-example shows that monotonicity of g_1, g_2 and *h* does not guarantee that the function G_1 will be monotonic, and thus, contrary to Dunn and Kirsner's claim, functional independence is *not* required to obtain a RA, even when all three functions are monotonic. Since the relationship between two tasks will be non-monotonic whenever one of G_1 or G_2 is non-monotonic, this example shows that both models 5 and 7 are consistent with the observation of an RA. We note that model 3 is the only two-process model that is not consistent with RA under assumption 4. This is because, in model 3, we can rewrite $t_1 = g_1(p_1) = g_1(h(p_2)) = G_1(p_2)$, and if *h* is monotonic and both g_i are monotonic, then G_1 will be monotonic regardless of the direction of the monotonicities.

Note that if an experimenter *a priori* believes in a process-pure model, then assumption 4 is sufficient to test between the f.d. versus f.i. versions of this model. Thus, in this case, if the

[3] The argument employs a specific counter-example, but note that many functions will constitute a counter-example, provided only that *h* is monotonic in a direction opposite to g_1 and g_2, and that the functions do not have constant slope.

experimenter observes an RA then she can infer that the two processes are *functionally independent*. However, process purity may not hold in many contexts, and thus inference of functional independence is not automatically warranted (Shallice, 1988; Jacoby, 1991).

So far we have established that under assumption 4 (or under less restricted assumptions), two-process models 2, 4, 5, 6 and 7 are all consistent with the observation of RA. We further investigated how the models fare under yet stricter assumptions about functional properties. We observed that of all assumptions that we have considered, only the very restricted assumption that g_1, g_2, and h are all monotonic in the same direction (assumption 6) is strict enough to reject all two-process models with f.d. processes. Note, however, that under assumption 6 an RA is no longer a necessary condition to reject the single process model. Namely, if assumption 6 is true then the sole observation of a negative association (i.e., a *crossed* or *cross-over double dissociation*, c.f. Dunn & Kirsner, this issue) would suffice to reject the single process model.

Finally, we briefly comment on the distinction between single-process and two-process models with f.d. processes. As noted earlier, a two-process model with f.d. processes can be rewritten in the form of a single process model. This observation led Dunn and Kirsner (1988) to conclude that such two-process models are indistinguishable from a single process model. We think the distinction is nevertheless meaningful. First of all, we have shown that two-process models with f.d. processes *can* be distinguished from the single process model (even under assumption 5; see Table 1). Secondly, it seems problematic and unnecessary to claim that there only exist mental processes that are functionally independent of each other. Even if two processes, p_2 and p_1, are functionally related by $h(p_2) = p_1$, does not the mere fact that $p_1 \neq p_2$ mean that there exist two different processes, viz., p_2 and $h(p_2)$?

Our contribution here is a finer grained analysis of how monotonicity plays a role in the DK framework, specifically with respect to drawing inferences from an RA. We have shown that explicit examination of assumptions of monotonicity in the DK framework can yield useful insights. Since it is likely that complex systems, like the brain, may recruit processes in a variety of ways (e.g., f.i. or f.d.; process pure or shared; monotonically related in the same or opposite direction), it is prudent to consider properties of different classes of models, as we do here.

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References

Dunn, J. C., & Kirsner, K. (this issue). What can we infer from double dissociations?

Dunn, J. C., & Kirsner, K. (1988). Discovering functionally independent mental processes: The principle of reversed association. *Psychological Review, 95*, 91-101.

Jacoby, L.L. (1991). A process dissociation framework: Separating automatic from intentional uses of memory. *Journal of Memory & Language*, *30*, 513-541.

Shallice, T. (1988). *From neuropsychology to mental structure.* Cambridge, UK: Cambridge University Press.