Parameterized Complexity and Bayesian Models

Johan Kwisthout and Iris van Rooij
Donders Institute for Brain, Cognition and Behaviour
Radboud University Nijmegen
{j.kwisthout,i.vanrooij}@donders.ru.nl

April 29, 2013

Bayesian models are becoming more and more popular. Their use as an engineering tool (e.g., in decision support systems or as classifiers) has been widespread already since the 1990s, but in recent years Bayesian models have enjoyed an enormous popularity in cognitive science as well. In the latter domain, Bayesian techniques are being used to model mental processes that underly cognitive behavior, either at the 'brain' level or at the 'mind' level. Yet, such models raise a theoretical challenge for explaining the computational efficiency of human brains: The NP-hardness of general Bayesian inference seems at odds with cognitive brain processing in *practice*, which occurs on a timescale of seconds or even milliseconds. In our research, we use parameterized complexity analysis to systematically address this challenge.

By and large, few parameterized complexity results are known for computational problems involving Bayesian networks. Fixed-parameter tractability of computing a posterior *probability* has been established *avant la lettre* for bounded treewidth (t) of the underlying graph and bounded cardinality (c) of the stochastic variables [7]. Computing the mode of a posterior distribution has a $O(c^{\frac{(\log(p)}{\log(1-p)}} \cdot c^t \cdot n)$ algorithm, where p denotes the probability of the mode of that distribution [2, 5]; it is para-NP-hard for $\{t, c\}$ alone [3] and $\{p, c\}$ alone [5]. Deciding whether the conditional probabilities of a network can be tuned to match some constraint on the distribution of a variable of interest is NP-hard in general, para-NP-hard for bounded treewidth, and W[1]-hard for the number of tunable conditional probabilities as parameter [4]. Learning the structure of a Bayesian network from data is NP-hard in general, but is fixed-parameter tractable for parameter set $\{t, d\}$, where t denotes the treewidth and d denotes the maximum degree of the super-structure [8]. These complexity results and others have been used to constrain computational models of cognition [1, 9].

For the *approximate* case there are even fewer parameterized complexity results. Such results would be particularly useful in those cases where the exact Bayesian computation is not fixed-parameter tractable for a set of parameters, but the corresponding approximate Bayesian computation is fixed-parameter tractable. One such example is given by the so-called *Most Simple Explanation* problem [6]. We showed that this problem is NP-hard to compute both exactly and approximately, but is fixed-parameter tractable to approximate (but not to compute exactly) for a particular parameter constraining the probability distribution (see [6] for details).

To summarize, given the evident import of parameterized complexity results for Bayesian models in domains such as cognitive science, and the paucity of such results so far, there are many opportunities to be explored. This is especially true in the case of approximate Bayesian computations. That few complexity results are known to date for approximate Bayesian computation is surprising, given the apparent success of Bayesian approximation algorithms in practical situations. Explaining why and under which circumstances such algorithms work well would constitute a major scientific advance with important applications in multiple domains of inquiry.

References

[1] Blokpoel, M., Kwisthout, J., van der Weide, T.P., Wareham, T., and van Rooij, I. (2013). A computational-level explanation of the speed of goal inference. To appear in *Journal of Mathematical Psychology.*

[2] Bodlaender, H. L., van den Eijkhof, F., and Van der Gaag, L. C. (2002). *On the complexity of the MPA problem in probabilistic networks.* In van Harmelen, F. (Ed.), *Proceedings of the 15th European Conference on Artificial Intelligence*, pp. 675–679.

[3] De Campos, C.P. (2011). *New complexity results for MAP in Bayesian networks.* In Walsh, T. (Ed.), *Proceedings of the 22nd International Joint Conference on Artificial Intelligence*, pp. 2100–2106.

[4] Kwisthout, J. (2009). *The computational complexity of probabilistic networks.* Ph.D. thesis, University of Utrecht.

[5] Kwisthout, J. (2011). Most probable explanations in Bayesian networks: Complexity and tractability. *International Journal of Approximate Reasoning*, *52*, 1452–1469.

[6] Kwisthout, J. and van Rooij, I. (2013). Bridging the gap between theory and practice of approximate Bayesian inference. *Cognitive Systems Research*, *24*, 2–8.

[7] Lauritzen, S.L. and Spiegelhalter, D.J. (1988). Local computations with probabilities on graphical structures and their application to expert systems. *Journal of the Royal Statistical Society. Series B (Methodological)*, *50*(2), 157–224.

[8] Ordyniak, S. and Szeider, S. (2013). Parameterized Complexity Results for Exact Bayesian Network Structure Learning. *Journal of Artificial Intelligence Research*, *46*, 263–302.

[9] Van Rooij, I., Kwisthout, J., Blokpoel, M., Szymanik, J., Wareham, T., and Toni, I. (2011). Intentional communication: Computationally easy or difficult? *Frontiers in Human Neuroscience*, *5*(52), 1–18.