Towards Neuromorphic Complexity Analysis

Johan Kwisthout
Donders Institute for Brain, Cognition and Behaviour
Radboud University Nijmegen, The Netherlands
email: j.kwisthout@donders.ru.nl

November 15, 2017

1 Introduction

Computational complexity theory offers an indication of the resources needed for a particular computational problem to be solved, as a function of the input size of a problem. These resources – most notably, time and memory – are typically fairly coarse and built on a theoretical abstract model of computation: Turing machines. Here, the 'time' resource refers to the number of state transitions in the machine, and the 'memory' resource refers to the number of memory cells on the tape that are used. This traditional way of representing computations may not be the most usable to describe information processing in the brain [BF16]. Most significantly, this model assumes symmetry in states and symbols, e.g., '0' and '1' are in principle interchangeable. This is in contrast to the power-efficient spike signals used in cortical processing; here, there is a clear asynchrony between the presence and absence of spikes: power-efficient coding requires as few spikes as possible. It has been proposed by a working group at the Dagstuhl seminar on Resource-Bounded Problem Solving (seminar 14341, [HvRVW14, p. 66]) to have a more refined, brain-focused model of computation in the brain, based on networks of spiking neurons, and have complexity measures loosely based on brain resources, such as spiking rates, network size and connectivity [Maa00, Maa14]. In this extended abstract we describe work-in-progress and future work towards that goal.

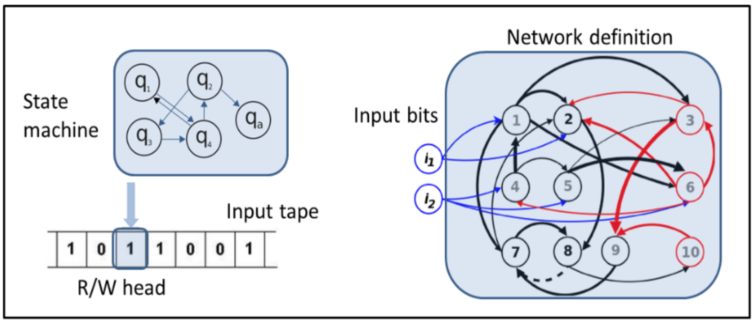

Figure 1: *Example of a Turing machine (left panel) and a network of spiking neurons (right panel). The Turing machine has an input (tape with symbols), a notion of computation (state machine transitions), resources (length of tape used and number of transitions), and a notion of acceptance (accepting state q_a). Our goal in this project is to construct similar notions in a network of spiking neurons.*

---

2 Structural complexity

In recent work we introduced complexity classes that capture the notion of resource-bounded stochastic computations, in particular approximate Bayesian inferences [Kwi18]. We show under which constraints such approximate inferences can be tractable, respectably remain NP-hard. These complexity classes are still based on the Turing machine model, and while appropriate to capture tractability and intractability of problems on traditional hardware, these classes are less suitable to capture the specific aspects of neuromorphic computing (such as power efficiency). For example, it is known that networks of spiking neurons can in principle implement any approximate Bayesian inference [BBNM11], but it is not known when such implementations are still tractable. We are currently working on a spiking neuron-based abstract neuromorphic model of computation and a basic notion of complexity and completeness in classes based on this model (Figure 1). Here we discuss a few 'parameters' we are currently investigating.

Acceptance criteria. For Turing Machines, different acceptance criteria lead to different complexity classes, characterizing different sorts of problems. For example, the class PP requires that a *yes*-instance of a problem is accepted by strictly more than half of the computation paths of a non-deterministic Turing machine. For the class BPP, this majority is qualified to be polynomially bounded away from $1/2$. This has huge imapct as PP-complete problems are highly intractable while BPP-problems are feasible. We currently investigate acceptance notions based to the stability of the distribution, assuming that differences here can lead to qualitatively different classes of problems.

Resources. In addition to the traditional resources 'time' and 'memory cells' (that can be translated to convergence time to a stationary distribution and the number of neurons) we investigate the resouces 'energy', that materializes as the spike-no spike proportion. We hypothesize that there exist 'energy-hard' problems that cannot be solved with limited energy, and also that 'time' and 'energy' trade-off in non-linear ways.

Completeness, reductions, and structural results. Defining complexity classes is rather useless without a notion of 'completeness' (what characterizes that hardest problems within a class), a means to reduce one problem into another problem while maintaining key properties such as time and energy constraints, and to relate these classes to traditional complexity classes and results.

3 Applications

Neuromorphic engineering [I$^+$11] typically addresses two goals: One the one hand, allowing for a better understanding of (resource-bounded) information processing in the human brain; on the other hand, using insights from neuronal implementations of said information processing to pave the way for the next generation of general-purpose computing. In this new field, emphasis is typically put on hardware development and algorithm design. We propose to augment this with a more fundamental formal study of the potential power *and inherent limitations* of neuromorphic computation, allowing for a) a better understanding of the capacity of the human brain, given its available resources, and b) understanding what can and cannot be done tractably on this new computing platform, on a much more fine-grained level compared to what is possible with traditional complexity theory.

References

[BBNM11] L. Buesing, J. Bill, B. Nessler, and W. Maass. Neural dynamics as sampling: A model for stochastic computation in recurrent networks of spiking neurons. *PLoS Computational Biology*, 7(11):e1002211, 2011.

[BF16] M.K. Benna and S. Fusi. Computational principles of synaptic memory consolidation. *Nature Neuroscience*, 19(12):1697–1706, 2016.

[HvRVW14] Y. Haxhimusa, I. van Rooij, S. Varma, and H.T. Wareham. Resource-bounded problem solving (dagstuhl seminar 14341). In *Dagstuhl Reports*, volume 4(8), 2014.

[I$^+$11] G. Indiveri et al. Neuromorphic silicon neuron circuits. *Frontiers in Neuroscience*, 5:e73, 2011.

[Kwi18] J. Kwisthout. Approximate inference in Bayesian networks: Parameterized complexity results. *International Journal of Approximate Reasoning*, 93:119–131, 2018.

[Maa00] W. Maass. Neural computation: a research topic for theoretical computer science? some thoughts and pointers. In *Bulletin of the European Association for Theoretical Computer Science (EATCS)*, volume 72, pages 149–158, 2000.

[Maa14] W. Maass. Noise as a resource for computation and learning in networks of spiking neurons. *Proceedings of the IEEE*, 102(5):860–880, 2014.