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Integrating global and local aspects of visual occlusion

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Abstract The phenomenon of visual occlusion has frequently been studied by means of two-dimensional line drawings. These drawings may elicit various interpretations. Sometimes a mosaic of shapes is seen, sometimes a shape that partly occludes another shape. In the latter case, observers often have a clear idea about the form of the partly occluded shape. Local and global pattern aspects both seem to be decisive with respect to the preferred interpretation. An attempt is made to integrate these aspects by applying the global-minimum principle to the perceptual complexity of three distinct components of those pattern interpretations: (i) The internal structure, dealing with each of the shapes separately, (ii) the external structure, dealing with the positional relation between these shapes, and (iii) the virtual structure, dealing with the occluded parts of the shapes. The perceptual complexity of each of these three components can be expressed in terms of structural information. The hypothesis that the perceptually preferred interpretation is the one for which the total information load is minimal is tested on many patterns stemming from different studies on pattern completion.

1 Introduction

Visual occlusion is a rule rather than an exception. In everyday life we mostly see parts of objects, yet experience them as being complete. The phenomenon of occlusion can be evoked by a simple two-dimensional line pattern, as in figure 1. This pattern can be interpreted as a rectangle and an L-shaped form, but most observers have a strong preference for the interpretation in which a rectangle is positioned in front of a square.

One might address two questions here. First, is completion of the background shape the result of a perceptual process or is it mainly based on cognitive processes? Second, what is the form of the background shape after completion? Gerbino and Salmaso (1987) and Sekuler and Palmer (1992) have made different attempts to answer the first question. Using a simultaneous-matching paradigm, Gerbino and Salmaso concluded that partly occluded shapes are functionally equivalent to complete shapes. In their simultaneous-matching task, the performance of subjects on partly occluded shapes did not differ from their performance on complete shapes. Sekuler and Palmer applied the so-called primed-matching paradigm to occlusion patterns. They concluded that completion of the background shape gradually develops

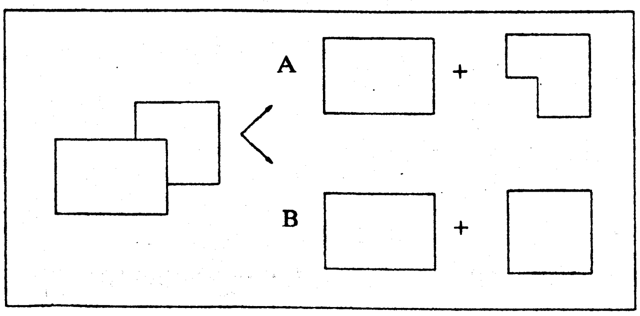


Figure 1. Two possible interpretations of one pattern. The occlusion interpretation (B) is preferred to the mosaic interpretation (A).

over time and that within a small amount of time, as short as 200 ms, partly occluded shapes are represented perceptually as complete shapes.

Taking these experimental results as a strong indication of the perceptual relevance of occlusion phenomena, we will focus on the second question: the form of the completed background shape. Several studies on occlusion patterns have been done by various researchers (cf Dinnerstein and Wertheimer 1957; Michotte et al 1964; Buffart et al 1981; Rock 1983; Boselie 1988; Boselie and Wouterlood 1989; Kellman and Shipley 1991). A major distinction between the approaches of these authors is the distinction between so-called local and global theories. Stressing the influence of local cues, Kanizsa (1975, 1985, 1986) emphasised the role of good continuation in perceptual organization. Indeed, the preferred interpretation in figure 1 (interpretation B) is predicted correctly on the basis of the good-continuation principle. This principle has been generalised and formalised by Kellman and Shipley (1991). They argued that completion will be such that the background shape is completed with the simplest continuous function between points of occlusion. Wouterlood and Boselie (1992) proposed a theory of visual completion that is completely based on local good continuation. However, their theory is designed to predict completions for a restricted set of shapes, ie shapes in which no regularities occur. Rock (1983) argued that the perceptual system avoids solutions with so-called unexplained regularities such as a coincidental meshing of borders. Despite the fact that Rock did not give a metric of coincidence, it is plausible that, according to Rock, the preferred completion in figure 1 (interpretation B) is less coincidental than the alternative completion (interpretation A) in which a rectangle fits exactly in an L-shape.

In contrast to local theories, global theories take into account regularities of the whole pattern. Early Gestalt psychologists (Kohler 1920; Wertheimer 1923) had indicated that perception tends to result in interpretations characterised by phenomenal simplicity and regularity. The gestalt law of pragnanz (Koffka 1935) can be considered as the precursor of the global-minimum principle (Hochberg and McAlister 1953), which states that the simplest interpretation of a pattern will be preferred. In the past, several descriptive models of visual patterns have been proposed (cf Hochberg and McAlister 1953; Leeuwenberg 1969, 1971; Simon 1972; Restle 1979). We will focus here on the structural information theory (SIT) which combines the global-minimum principle with Leeuwenberg's coding model for visual patterns. Later on, we will give a brief account of the coding model as elaborated by Van der Helm and Leeuwenberg (1991). This global approach also predicts that interpretation B of figure 1 is preferred. The two shapes of this interpretation are more regular than are the shapes of interpretation A. However, there are patterns for which local and global theories predict different interpretations. A demonstration is given in figure 2.

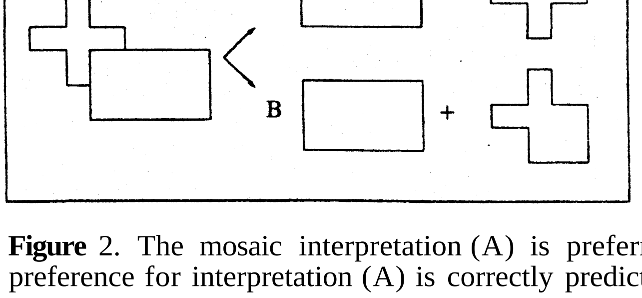


Figure 2. The mosaic interpretation (A) is preferred to the occlusion interpretation (B). The preference for interpretation (A) is correctly predicted by the global minimum principle.

For figure 2, the global-minimum principle correctly predicts that most subjects have a strong preference for interpretation A: a mosaic with a cross juxtaposed onto a rectangle. On the basis of the good-continuation principle, however, a preference for interpretation B would be predicted. Rock's coincidence-explanation principle does not lead unambiguously to either one of the two solutions. On the one hand, interpretation B avoids a coincidental meshing of borders while, on the other hand, it leaves unexplained the coincidence of the symmetry of the visible cross.

In spite of the initial success of the global-minimum principle, patterns have been constructed (Rock 1983; Kanizsa 1985) that seem to contradict predictions made on the basis of this principle. Using the perceptual coding system within SIT, Boselie and Leeuwenberg (1986) were initially able to refute a number of counter examples that were brought in against the global-minimum principle. However, in two other studies, Boselie (1988) and Boselie and Wouterlood (1989) constructed and tested a great number of occlusion patterns, and found results in contradiction with predictions made by SIT. Figure 3 is taken from Boselie and Wouterlood (1989).

In figure 3, the two shapes in interpretation A clearly are more regular and therefore simpler than are the shapes in interpretation B. Yet, in a task in which subjects had to draw the contours of the perceived completion, none of the subjects showed a preference for interpretation A, whereas most of the subjects preferred interpretation B. Boselie and Wouterlood rightly remarked that the results do not necessarily imply a rejection of the global-minimum principle as such. This is in line with the work of Hatfield and Epstein (1985) who stated that a straightforward test of the global minimum principle is not possible. The principle relies heavily on assumptions about what simplicity actually is. Yet, confronted with a large number of patterns that seemed to defy the global minimum principle, Boselie (1988) suggested that locally simplest completions dominate in pattern completion. Furthermore, he stated that a globally simplest completion is made only in case of local ambiguity, ie when several locally simplest completions are possible. In any case, single factors such as familiarity, good continuation, local configurations, or coincidence appeared to be inadequate to explain the preferences of subjects.

What has to be done, in our view, is to examine both the global and the local aspects of occlusion patterns and determine their joint contribution to the complexity of pattern interpretations. With that basis, the global minimum principle operates as a criterion to select the most preferred interpretation. First, however, we need to be clear about what the complexity of an interpretation actually refers to.

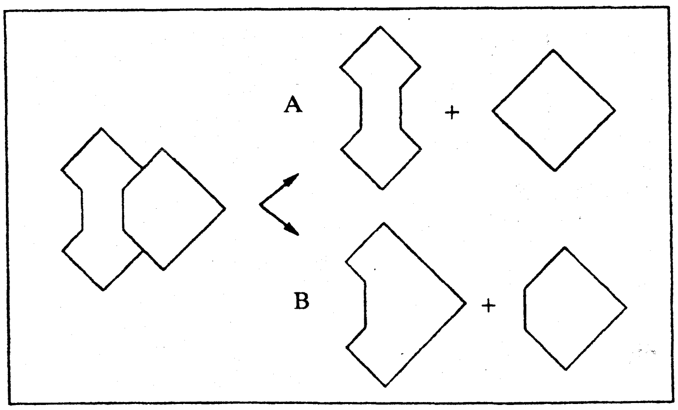


Figure 3. Interpretation B is preferred to interpretation A. This preference is not in line with the global-minimum principle as specified thus far.

2 Perceptual complexity

We start with a distinction between two kinds of complexities. In order to illustrate this, we consider an analogy from the domain of problem solving. Problems may be difficult to solve. Yet it is common experience that a final solution may be astonishingly simple. Evidently, unravelling a problem is not the same thing as describing its solution. In our view, an analogous distinction is relevant for visual occlusion. That is, a distinction can be made between the acquisition of the preferred interpretation, and the interpretation itself as it might be stored in memory. Therefore, we make a distinction between the **'perceptual complexity'** and the **'memory complexity'** of an interpretation. The perceptual complexity refers to the complexity of an interpretation in relation to the pattern, whereas the memory complexity of an interpretation is determined by the memory space required to store that interpretation. To a certain extent, the proposed distinction corresponds with the distinction between phenomenal simplicity and descriptive economy made by Hatfield and Epstein (1985). However, the term phenomenal simplicity generally expresses a tendency to simplicity of shape. As we will see, in the case of occlusion patterns, simplicity of shape is just one aspect of the perceptual complexity of a certain interpretation. The notion of descriptive economy comes close to what we have called memory complexity. Nevertheless, we prefer the latter term because, as will be demonstrated, the determination of the perceptual complexity also proceeds in a descriptive way. We argue that, in order to predict preferred interpretations on the basis of complexity, the perceptual complexity has to be considered. In the remainder of this paper, the perceptual complexity of an interpretation will be discussed in more detail.

We will consider three components in visual occlusion that determine the perceptual complexity of an interpretation: shape, position, and occlusion. These components are embedded in three different types of structure, termed the **'internal structure'**, the **'external structure'**, and the **'virtual structure'**, respectively (see figure 4). We will clarify these three concepts and show that each of them independently affects the preference for perceptual interpretations of occlusion patterns. Furthermore, it will be argued that the sum of the perceptual complexities of the three structures determines the

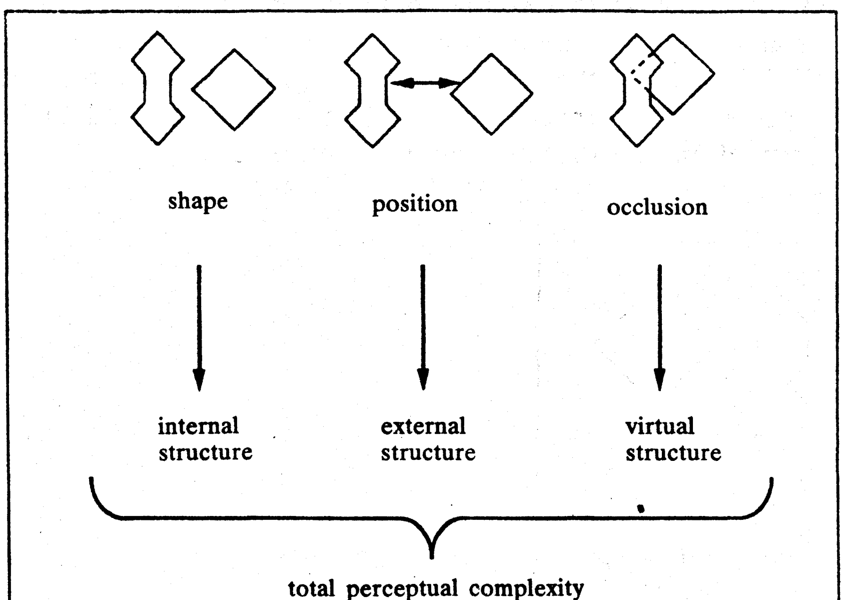


Figure 4. Three components, shape, position, and occlusion, are reflected by three types of structure. The total perceptual complexity of a pattern interpretation is constituted by the sum of the perceptual complexities of the three structures.

perceptual complexity of an interpretation. Finally, applying the global-minimum principle, we hypothesise that the total perceptual complexity of the most-preferred interpretation is lower than that of any other interpretation.

As the quantification of the perceptual complexities of the three structures proceeds in terms of structural information, we first introduce the concept of structural information.

3 Structural information

We explicate the notion of structural information by considering symbol series. By means of a set of coding rules (van der Helm and Leeuwenberg 1991) regularity is extracted from such a series. In the structural-information model only three classes of regularity play a role, namely iterations, symmetries, and alternations. Each of these three classes can be described by a corresponding coding rule. In table 1, an example is given for each of the three coding rules: The symbol series 'aaa' is encoded by means of the iteration rule and results in the code 3 x (a), the series 'abcba' is encoded by the symmetry rule, resulting in the code S[(a)(b),(c)], and the series 'abac' is encoded by the alternation rule, resulting in the code ⟨(a)⟩/⟨(b)(c)⟩.

Each code comprises a certain amount of structural information. Now, given a specific symbol series, the aim is to find a minimum code, ie a code with a minimal amount of information. We will briefly sketch the way in which the amount of structural information is determined. Detailed discussion on this topic can be found elsewhere (van der Helm and Leeuwenberg, 1991; van der Helm et al 1992). In van der Helm et al (1992) a new metric of structural information was proposed and tested on symbol series. This information metric takes into account all parameters at all hierarchical levels in the code. The number of parameters in the code of a symbol series determines the amount of irregularity in the series according to that code. The amount of structural information therefore reflects this amount of irregularity. Applying this notion to the codes in table 1, the information loads are 1 (because of parameter 'a'), 3 (parameters 'a', 'b', and 'c'), and 3 (parameters 'a', 'b', and 'c'), respectively. Note that coding operations, reflected by, for instance, the 'S'-symbol in a code, are not considered as an information load. Note further that each code in table 1 contains only one hierarchical level. In order to provide some examples of a hierarchical code consider the minimum codes of the symbol series 'ababab' and 'abcab'. The minimum code of the series 'ababab' is given by 3 x (ab). The information load of this code equals 3, constituted by the parameters 'a' and 'b', and the group 'ab', which is treated as one entity as it is in fact a parameter at a higher level in the code. The encoding of 'abcab' into S[(ab),(c)] yields an information load of 4, constituted by the parameters 'a', 'b, 'c', and the group 'ab'. In contrast to this code, one may consider the encoding of 'abcba' into S[(a)(b),(c)], which has only one hierarchical level and an information load of 3.

Table 1. The three types of regularity employed in SIT. Encoding of a symbol series proceeds by extracting these regularities from the series. The information load, /, of the code reflects the amount of irregularity. Each parameter that contributes to the information load is indicated with a dot.

Regularity	Symbol series	Code	/
Iteration	aaa	$3 \times (\underset{\cdot}{a})$	1
Symmetry	abcba	$S[(\underset{\cdot}{a})(\underset{\cdot}{b}),(\underset{\cdot}{c})]$	3
Alternation	abac	$\langle(\underset{\cdot}{a})\rangle/\langle(\underset{\cdot}{b})(\underset{\cdot}{c})\rangle$	3

The encoding, from the initial symbol series to the selection of the code with the lowest information load, is completely processed by the computer program PISA (van der Helm, 1988). There is an essential conceptual difference between metrics of information that were applied in the past and the new-information metric mentioned above. In early versions of SIT (Leeuwenberg 1969, 1971; Buffart et al 1981) the required memory space to store a code was considered as the basis for their information metric. This old information metric, therefore, conceptually fits in with the notion of memory complexity. In the concept of the new information metric, however, this memory argument has been rejected and replaced by the irregularity argument described above. This irregularity argument emphasises the relationship between code and symbol series. Therefore, the new information fits in with the notion of perceptual complexity. The quantification of the perceptual complexity of the interpretations of occlusion patterns by means of structural information is possible after operationalising the perceptual complexities of the three structures that are involved. We start with the internal structure.

4 The internal structure

The internal structure of an interpretation involves the shapes within that interpretation. Accordingly, its perceptual complexity is constituted by the complexities of these shapes. To determine the complexity of a shape, the shape is represented by a symbol series. Every contour element, line, or angle, is represented by a symbol such that equal elements are represented by equal symbols. The order of appearance of sequential elements on the contour is preserved in the order of the respective symbols in the series that represents the shape. This series is called the primitive code of the pattern. The encoding of the symbol series proceeds cyclically, ie irrespective of the element on the contour that is represented by the first symbol. The information load of the minimum code determines the complexity of the shape. Thus, the total complexity of the internal structure of an interpretation is made up of the sum of the complexities of all shapes within that interpretation. In the following, the information load of the internal structure will be referred to as 7_{int}

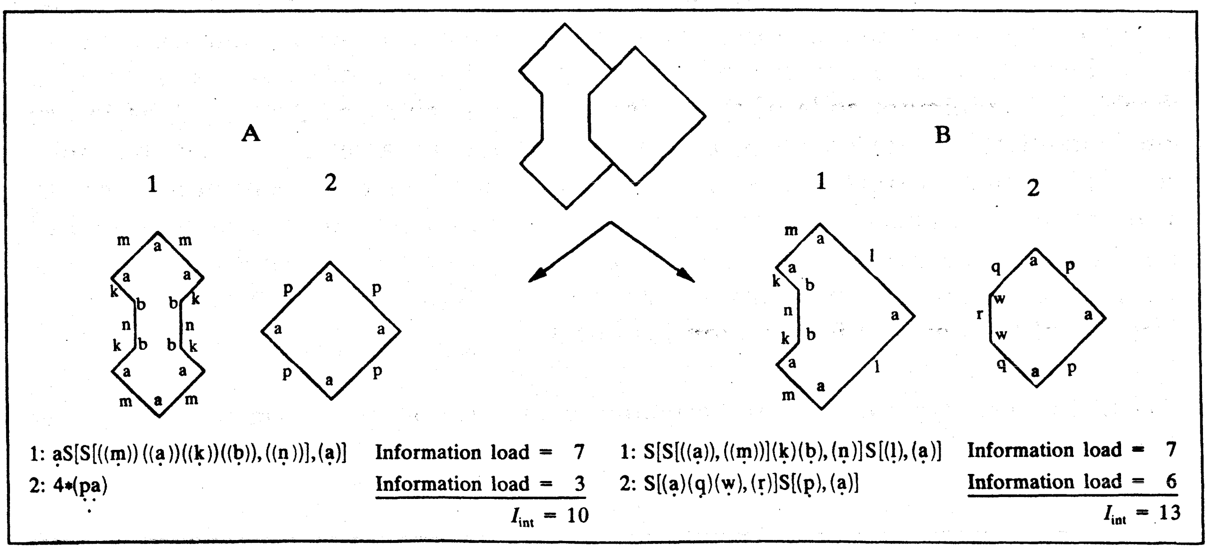


Figure 5. The calculation of I_{int} performed for two different interpretations of a pattern. First, each shape within an interpretation is represented by a symbol series such that all lines and angles are represented each by one symbol, and such that their order and identity are preserved. Second, each symbol series is encoded, resulting in a minimum code. The information load for each code equals the number of descriptive parameters in the code (indicated by a dot). Third, I_{int} of the interpretation is calculated by summing the information loads of the minimal codes.

In figure 5, the minimum codes and their information loads are presented for the shapes within both interpretations of the patterns in figure 3. For example, within interpretation A, the second shape is a square represented by symbol series 'papapapa'. The minimal code of this series is obtained by applying the iteration rule, resulting in 4 x (pa). The amount of information is calculated by counting the total number of parameters, ie 'p', 'a', and 'pa': thus / = 3. The determination of the complexities of the other shapes proceeds in a similar way. Next, the total complexity of the internal structure for a specific interpretation is calculated by summing the complexities of the internal structure of both shapes within that interpretation. It appears that interpretation A has a lower I_{int} than does interpretation B (the values being 10 and 13, respectively). As interpretation B is preferred to interpretation A, it is clear that a 'simplest-shape' operationalisation of the minimum principle, based solely on the internal structure, leads to a wrong prediction.

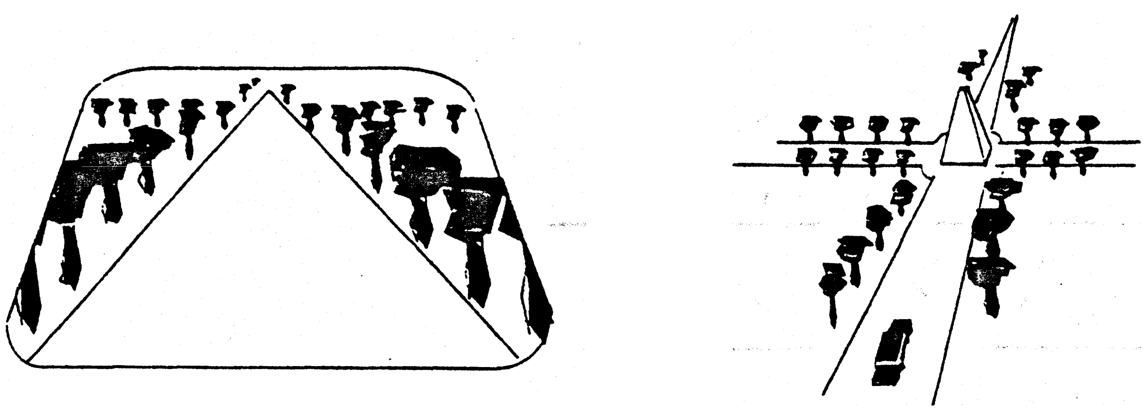
Once an occlusion pattern has been interpreted, and the perceived shapes have been memorised, the internal structure suffices. However, in interpreting the pattern, the perceptual complexities both of the external and of the virtual structure also play a role. We first take a closer look at the external structure. The section on the external structure is rather extensive, as it introduces some new concepts which require a more detailed discussion.

5 The external structure

The external structure of an interpretation applies to the positional relation between the shapes within that interpretation. It is well known that the relative position of objects affects perception. Let us consider an extraordinary case.

Imagine a somewhat misty road. In figure 6a a view of the road, as momentarily seen through the windscreen of a car, is depicted—the road seems to be straight ahead to the horizon. The real situation may be quite different, for instance including an obelisk, as depicted in figure 6b. Nevertheless, figure 6a will be interpreted as 'only a road' and not as 'road plus obelisk'. The reason for this interpretation clearly lies in the fact that, in figure 6a, the 'road plus obelisk' interpretation implies coinciding contours of the road and the obelisk, whereas the 'only a road' interpretation does not imply such coincidences. How should one understand the role of such coincidences, and how can their effect be quantified? We will discuss both issues successively.

According to the general viewpoint principle (cf Huffman 1971), the positions of the perceiver and the objects in the scene are assumed to be general, ie coincidences



(a) (b)

Figure 6. Two possible views of the same situation. In (a), a view through the windscreen of a car is depicted, whereas (b) may be a bird's eye view. Obviously, the point of view of the car driver induces a coincidental alignment of borders. Due to this alignment, the car driver's interpretation of the scene does not involve an obelisk.

are assumed not to be present in the momentary percept. In this way, the 'road plus obelisk' interpretation is not even considered as one of the possible interpretations. Similarly, but less rigorously, Rock (1983) argued that the perceptual system avoids interpretations which imply coincidences that are caused by unexplained regularities or co-occurrences. In this way, the 'road plus obelisk' interpretation may be considered as one of the possible interpretations, but is rejected in favour of the 'only a road' interpretation, since the former interpretation implies unexplained regularities (ie the coinciding contours of road and obelisk), whereas the latter interpretation does not. Note that Rock did not use the term 'probability' as such in order to explain these preferences, as probability also may include the influence of past experience. Nevertheless, in the following we will use the term probability, but solely in the context of regularity, thereby excluding past experience. Now, let us take a closer look at the role of coincidences by means of figure 7.

In figure 7, four random throws of two matches are depicted. Somehow, the configurations on the left look more probable than the configurations on the right. It is obvious, however, that each specific configuration has an equal probability of being the result of a random throw. The solution to this incongruity is that the kinds of configurations shown in figure 7 are not equally probable: among all possible configurations, there are more configurations like that in figure 7a than there are like that in figure 7d. For this reason supporters of the coincidence-explanation principle do not consider the probability of a specific configuration, but the probability of occurrence of a perceptually salient feature, such as the T-junction in figure 7b. This means that the probability of a specific configuration is based on the perceptual classification of that configuration. In this way, it can indeed be argued that different interpretations, like those mentioned above for the momentary percept in figure 6a, differ in probability and that the most probable one is selected. But then, two questions arise. First, how is it established to which class an interpretation belongs? Second, how is the probability for that class determined? Only if these two questions are answered might probability be acceptable as a basic explanatory principle.

In our view probability merely functions as a secondary concept. Our arguments are as follows. Given an interpretation of a momentary percept, coinciding positions

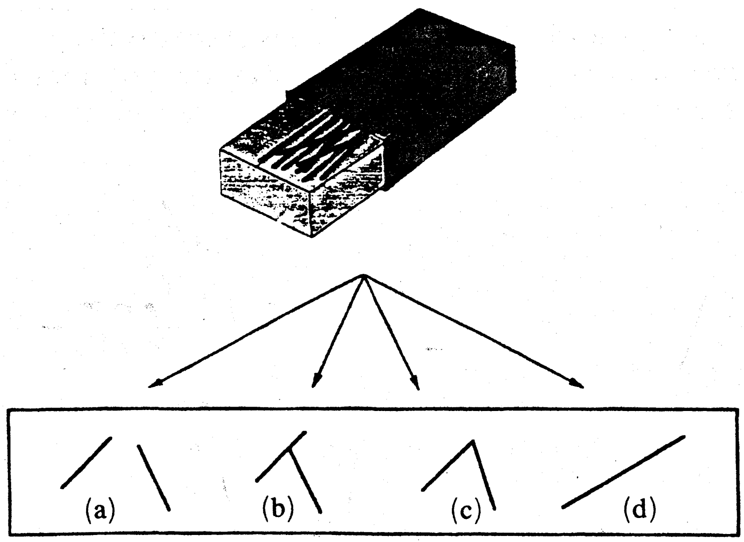


Figure 7. Four random throws of two matches are represented in (a-d). Most observers do not regard each of these configurations as equally probable. The configurations on the left look more probable than do the configurations on the right. The solution to this incongruity is that, as a result of a random throw, there are more configurations like those on the left than there are like those on the right.

are just regularities caused by the positional relation between the objects in the interpretation. So, just like the internal structure of an interpretation, the positional relation too is a source of regularities. Now in our approach, an interpretation is represented by a description and a quantification of the regularity in that interpretation, which implies classification. That is, the interpretation belongs to the class of all interpretations that contain precisely the same kinds of regularity and the same degree of regularity (Collard and Buffart 1983). Collard and Buffart also demonstrated that the greater the degree of regularity in an interpretation, the smaller the class to which that interpretation belongs. So, in fact, the probability of a specific interpretation could be related directly to the degree of regularity already quantified (ie the number of descriptive parameters) in that interpretation.

Note that our line of reasoning provides answers to the two questions raised above. Also note that it implies that any probability calculus based on the number of descriptive parameters reduces itself to a secondary concept (see also Leeuwenberg and Boselie 1988). That is, in our view, probability is not the basis for the perceptual role of coincidences, but merely a postperceptual side effect based on the perceptual role of regularities.

Now that we have explained our views of the role of coincidences or regularities in the positional relation between objects, we can turn to the quantification of their effects. In the following we will refer to this positional relation as the 'external structure' of an interpretation. On the one hand, regularities in the external structure of an interpretation would reduce the memory complexity of an interpretation, as they enable a simpler description of an already available interpretation. On the other hand, however, in interpreting a pattern, regularities in the external structure of an interpretation form a perceptual obstacle with respect to that interpretation. Perceptually, these regularities have a binding effect on the pattern parts that, within the interpretation, are considered to be independent objects. For instance, the pattern in figure 7d on its own is hardly interpreted as a concatenation of two independent objects (represented by line elements). Thus, coincidences enlarge the perceptual complexity of an interpretation, because the coinciding object parts have to be 'dissociated' perceptually. This dissociation eliminates the regularities in the external structure by adding the same amount of irregularities. The quantification of the perceptual complexity of the external structure of an interpretation is shown in figure 8.

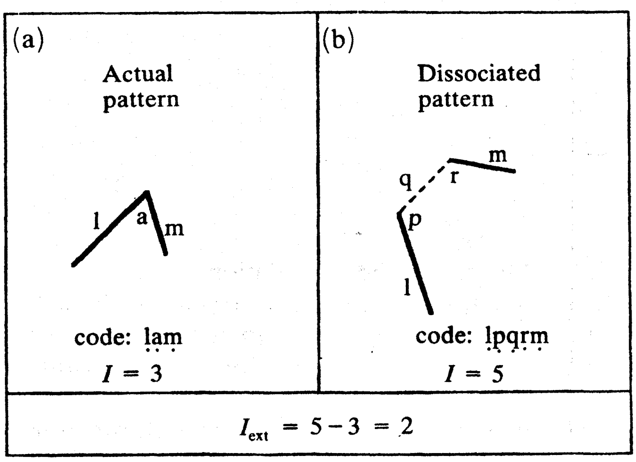


Figure 8. The hook pattern (a) may be interpreted as two independent sticks. The perceptual complexity of the external structure of this interpretation is quantified by subtracting the number of descriptive parameters needed to describe the actual pattern (a) from the number of descriptive parameters needed to describe the dissociated pattern (b). All descriptive elements are indicated with a dot.

Considering the hook-like pattern of figure 8a as one object, it can be described by three parameters 'l', 'a', and 'm' representing a line, an angle, and a line, respectively. Another interpretation may be two separate line-elements, representing for instance two sticks. In order to determine the perceptual complexity of the external structure of the latter interpretation, a dissociation of the sticks has to be established, such that there are no regularities left between them (see figure 8b). The pattern in figure 8b can be described by 5 parameters T, 'p', 'q', 'r', and 'm' representing a line, an angle, a distance between the lines (indicated by the dashed line), an angle, and a line, respectively. The difference in the number of descriptive parameters for the actual pattern (figure 8a) and the dissociated pattern (figure 8b) is entirely due to the variation in the relative position of the sticks in the two patterns. This difference $(5-3 = 2)$ therefore reflects the degree of irregularity, in terms of structural information, that has to be added to the actual pattern in order to establish the dissociation. Therefore, this amount of structural information is taken to be the perceptual complexity of the external structure of the 'two sticks' interpretation. The complexity of the external structure will be referred to as I_{ext}.

In the following we will apply this calculation strictly locally at each junction within a pattern. Consider for example the pattern in figure 9a. This pattern can easily be conceived as a juxtaposition of two triangles. In figure 9b a dissociation of these two triangles is depicted. The junction that constitutes the connection between the triangles in the actual pattern is shown in figure 9c. Note that, as the determination of I_{ext} proceeds locally at each junction, all lines at the junction have been drawn with an arbitrary length. Accordingly, all line elements have been labelled with different symbols. So, only the specific way in which they are connected is decisive with respect to I_{ext}. In figure 9d the junction has been dissociated, according to the dissociation of the two triangles. In order to calculate the complexity of the external structure, we will first determine the minimal number of descriptive parameters for the junction in figure 9c. The description proceeds by tracing the line elements of the

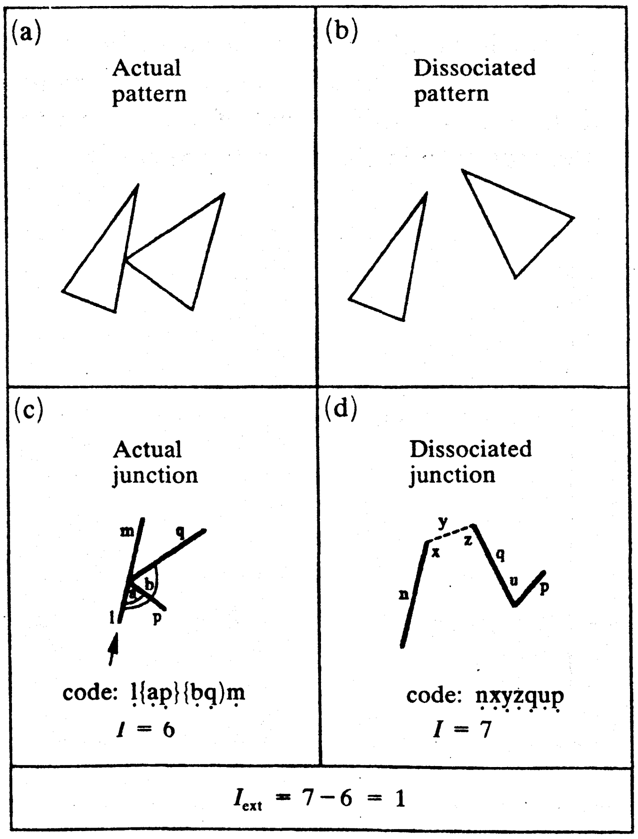


Figure 9. The calculation of I_{ext} for the junction between two triangles. The two triangles are shown joined (a) and dissociated (b). The junction that constitutes the connection between the two triangles in the actual pattern is also shown joined (c) and dissociated (d). The codes for the two junctions are shown below, and the information loads (descriptive parameters) indicated by dots. See text for details of calculation.

junction, thereby fixating their relative position. For example, the junction in figure 9c may be described as follows. Starting with line ‘l’ (see arrow in figure 9c), there is a bifurcation such that lines T and ‘p’ include an angle ‘a’; there is a second bifurcation where lines T and ‘q’ include an angle ‘b’, and finally, line T continues—without changing direction—into line ‘m’. This description enables one to reconstruct this particular junction. Note that all other angles in the junction are implicitly given in the posed description. So, a primitive code for that junction is l{ap}{bq}m [the brackets within the code express a bifurcation, cf Leeuwenberg (1971)]. This code, which cannot be reduced by coding rules, contains 6 parameters and therefore has an information load of 6. Other codes are possible, but would not result in a lower information load. We now turn to the dissociated junction (figure 9d). All lines and angles are labelled again, including the distance and angles between the contours. One parameter ‘n’ is required for the description of the junction part that stems from the edge of the left triangle, three parameters (‘q’, V, and ‘p’) are required for the junction part that stems from the corner of the right angle and three parameters (‘x’, ‘y’, and ‘z’) specify the relative position of the two dissociated junction parts. The corresponding code (nxyzqup) therefore contains 7 parameters. As in the example in figure 8, the complexity of the external structure is given by the difference between the complexity of the dissociated junction and the complexity of the actual junction. Therefore $I_{ext} = 7-6 = 1$.

In the following we will, for reasons of simplicity, only indicate the minimal number of parameters that is required to describe the junction and its dissociation. In the drawings, these parameters will be indicated by dots. In figure 10 the contribution to I_{ext} is determined for junctions of various patterns. For each pattern in the first column in figure 10, only one specific junction is considered—that junction is indicated with bold lines. The calculation of I_{ext} proceeds analogously with that in the previous examples.

So far, the contribution to the complexity of the external structure has been calculated for just one junction within a pattern. However, other junctions in a pattern might also increase I_{ext}. Let us consider figure 11, in which pattern 3 has been drawn once more. In this pattern three junctions appear, labelled A, B, and C. Junction A is the same as the one considered for pattern 3 in figure 10. So, junction A yields a contribution of 1 to I_{ext}. Junction B, in which a corner of a triangle adjoins an edge of another triangle, reveals the same situation as in figure 9, or as in pattern 2 of figure 10. Hence, junction B also yields a contribution of 1 to I_{ext}. Finally, the contribution of the crossing lines of junction C to I_{ext} is the same as for the junction considered for pattern 1 in figure 10. In that case the actual junction and the dissociated junction have the same information loads. Therefore, junction C yields a contribution of 0 to I_{ext}. Now the complexity of the external structure for the complete pattern interpretation (ie the two triangles) is calculated by taking the sum of these three contributions. So, $I_{ext} = 2$.

As the calculation of I_{ext} has now been set, we turn to occlusion patterns. Consider again the pattern in figure 3. This pattern and its interpretations, A and B, are depicted once more in figure 12. In order to visualise some dissociations of the shapes within each interpretation, small translations are applied in different directions. As a result of these small translations associated contour elements, that belong to different shapes, become dissociated in the generalised positions. Apparently the translations cause more drastic changes for interpretation A than for interpretation B. Therefore it can be expected that I_{ext} of interpretation A will be higher than I_{ext} of interpretation B. In figure 13, I_{ext} is determined for both interpretations of the pattern. We start with interpretation A. The junctions are labelled a1 to a4. Note that junctions a1 and a4 are exactly the same. This also holds for junctions a2

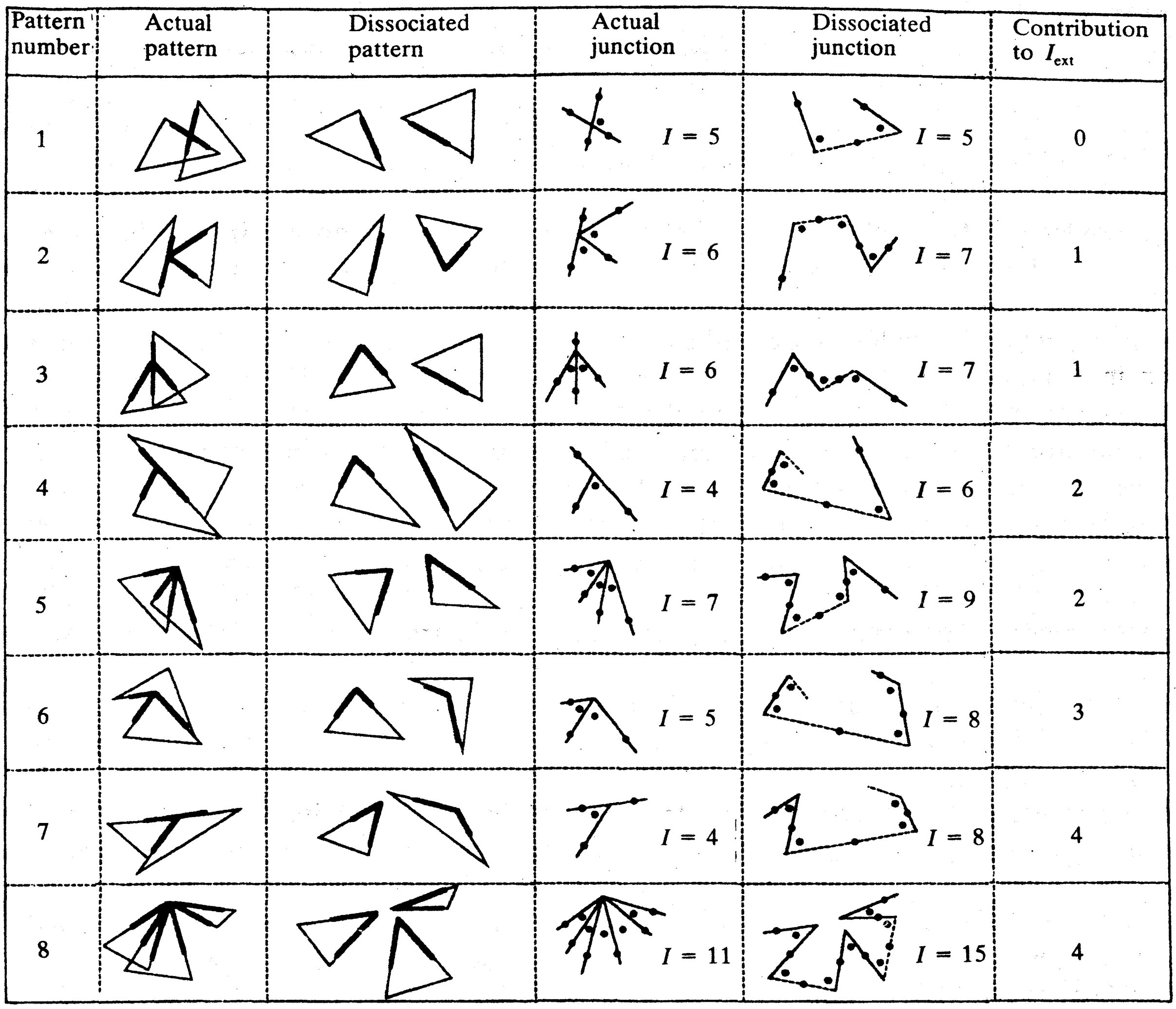


Figure 10. The calculation of the contribution to the complexity of the external structure of a pattern for various junctions that may arise between contours of shapes. The bold lines indicate the junction to be considered. In the two columns showing the actual and dissociated junctions the black dots indicate the minimal number of descriptive parameters needed to describe the junction.

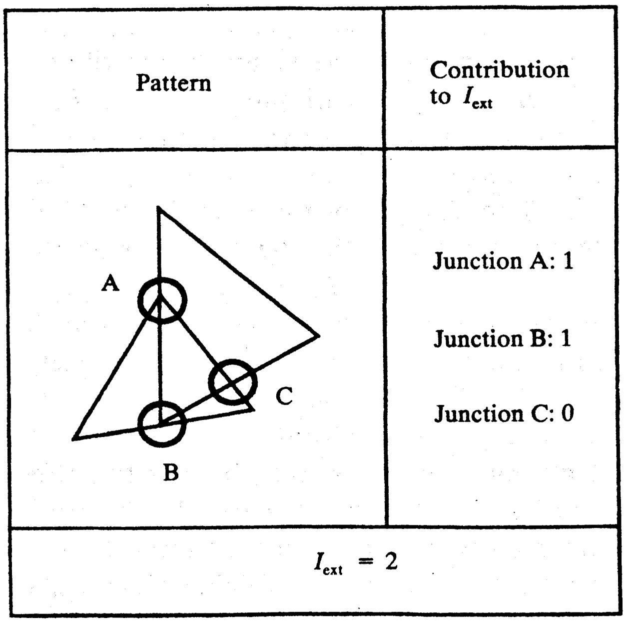


Figure 11. Pattern 3 of figure 10, enlarged. Three junctions appear, labelled A, B, and C. The assessment of the complexity of the external structure of this pattern proceeds by summing the separate contributions to the external structure for all junctions in the pattern.

and a3. The calculation of the contribution to I_{ext} for these junctions is shown below the junctions. Actually, the calculation is the same as performed for the junction of pattern 4 in figure 10, which yielded a contribution of 2 to I_{ext}. Hence, I_{ext}(interpretation A) = $4 \times 2 = 8$. Within interpretation B there are two identical junctions (labelled b1 and b2). The calculation for these junctions, shown below them, is the same as that performed for the junction of pattern 1 in figure 10, which yielded a contribution 0 to I_{ext}. Hence, I_{ext}(interpretation B) = 0.

The determination of the perceptual complexity of the external structure by taking the sum of the contributions to I_{ext} over all junctions in a pattern enables an unambiguous calculation of I_{ext}. One may argue that other relational properties between pattern and interpretation codetermine the complexity of the external structure. For instance, interpretation B in figure 3 induces a special position of shapes, as this position is such that the visible part of shape Bl is a nice regular shape, equivalent to shape A1. Now, should the regularity of the visible part of an occluded shape therefore increase I_{ext}? Our answer is no. The regularity of shape Al is already captured in the internal structure of interpretation A. Therefore, this regularity as such supports interpretation A, instead of suppressing interpretation B, so it does not have to be accounted for when considering interpretation B.

Summarising, the external structure deals with regularities between the objects, whereas the internal structure concerns regularities within the objects. In both cases, these regularities will bind pattern elements. The binding in the internal structure strengthens an interpretation, whereas the binding in the external structure weakens an interpretation. In addition to these two structures there is a third structure that plays a role, namely the virtual structure.

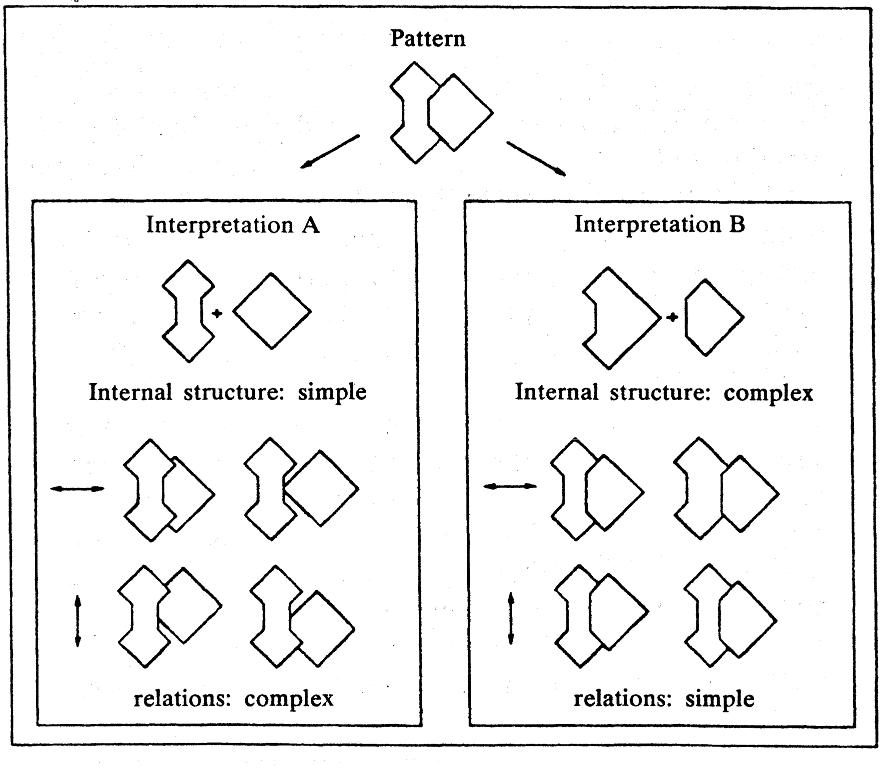


Figure 12. Dissociations of shapes for two different interpretations of one pattern. In interpretation A, the internal structure of the interpretation is simpler than in interpretation B, yet the dissociations for A cause a larger change in the positional relation of the shapes than do the dissociations for B.

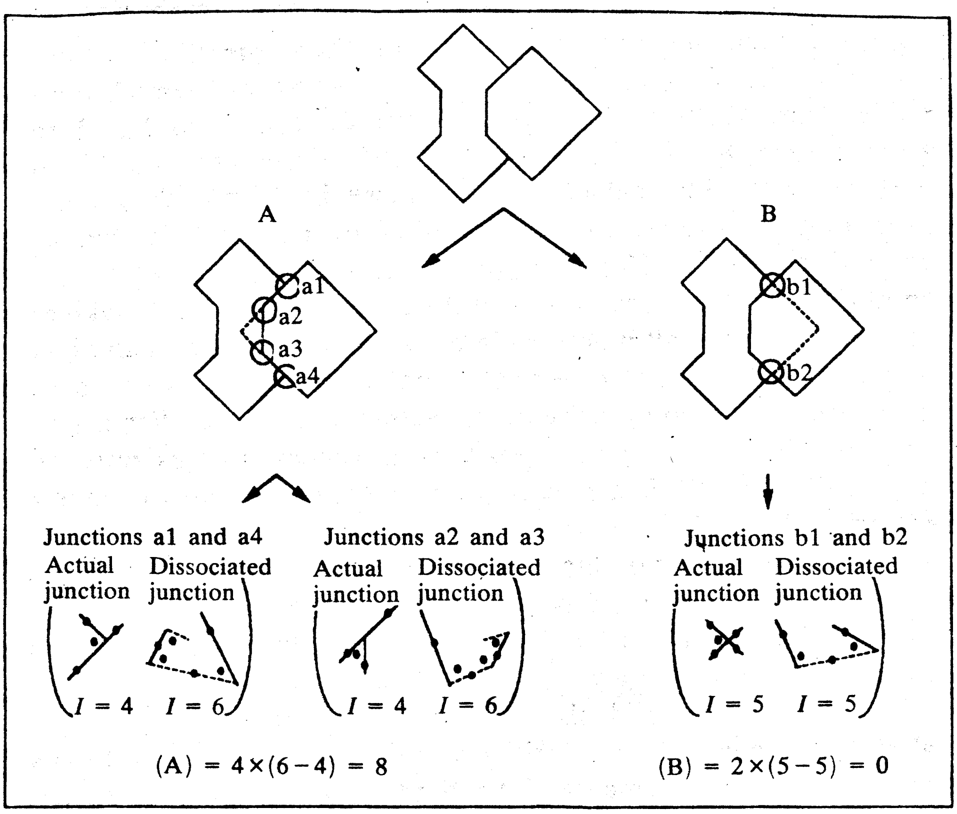


Figure 13. The calculation of the perceptual complexity of the external structure of two possible interpretations of one pattern. For each junction, the number of descriptive parameters for the actual position is subtracted from the number of descriptive parameters for the dissociated position. The perceptual complexity of the external structure for each interpretation is determined by the sum of these differences over all junctions.

6 The virtual structure

Whenever completions occur, there is a discrepancy between the number of pattern elements in the proximal stimulus and the number of elements within an interpretation. Virtual elements are added to complete the occluded shape. These virtual elements constitute the virtual structure of the interpretation. Note that for an already-available interpretation, the virtual elements are part of the shapes and therefore do not contribute to the memory complexity of that interpretation. Patterns can be made, however, in which several alternative interpretations have the same perceptual complexity both for the internal structure and for the external structure, but which evoke a clear preference for only one specific interpretation. Such a case is shown in figure 14. Interpretation A is preferred to interpretation B, presumably because of its relatively simple virtual structure. Therefore, the contribution of the virtual structure to the perceptual complexity of a pattern interpretation is considered to be independent of the contributions of the other structures.

To quantify the perceptual complexity of the virtual structure, we will make an intuitively appealing assumption: The perceptual complexity of an interpretation increases with the number of virtual elements, ie additional lines and angles. Our argument is that, before primitive codes of the internal structure can be reduced by means of encoding, the primitive virtual elements have to be available. Insofar as completion does not unify different visible line-elements into a single element (by means of continuation), the number of virtual elements is specified by the difference between the number of visible elements of the partly occluded shape and its total number of elements. In the following, the quantification of the perceptual complexity of the virtual structure will be referred to as I_{virt}. With respect to the pattern in figure 3, the complexity of the virtual structure is the same for both interpretations (see figure 15). In both interpretations, I_{virt} equals 1. Notice that this number is not necessarily equal to the number of elements of the occluded part. In figure 15, this part in both cases is a hook pattern containing 3 elements, but $I_{virt} = 1$. In fact, the definition of I_{virt} implies that continuation of a visible line behind an occluding shape does not introduce a new virtual element and therefore does not increase I_{virt}.

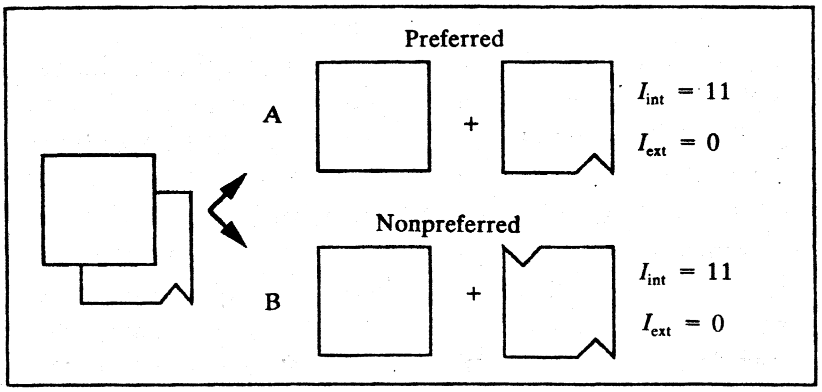


Figure 14. For the interpretations of the pattern on the left, both I_{int} and I_{ext} are equal. Yet, interpretation A is preferred to interpretation B, presumably due to the fact that the occluded part in A is relatively simple.

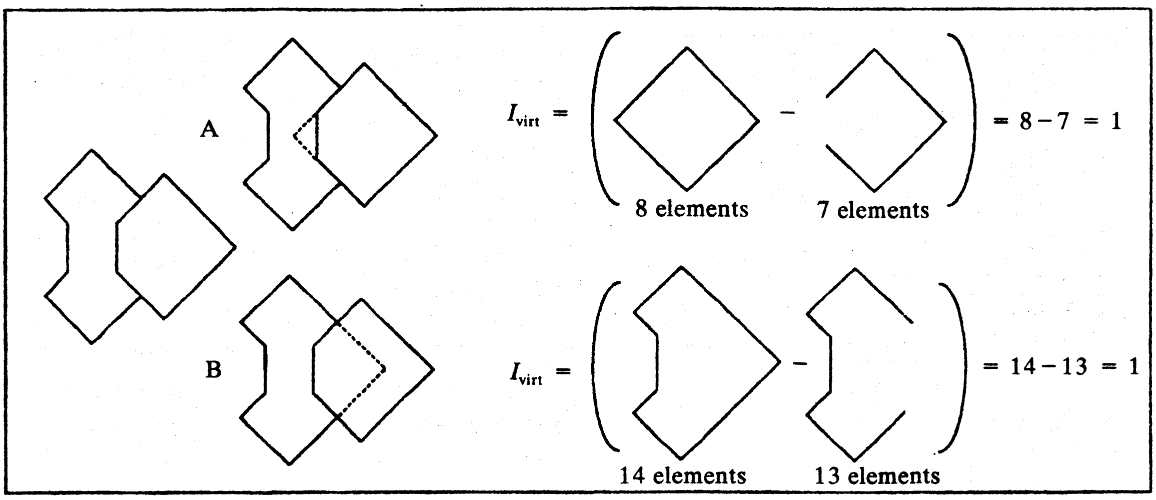


Figure 15. The perceptual complexity of the virtual structure of interpretations A and B is calculated by subtracting the number of visible elements of the partly occluded shape from its total number of elements.

7 Testing the theory

We define the total perceptual complexity of an occlusion interpretation as the sum of the perceptual complexities of the three types of structure:

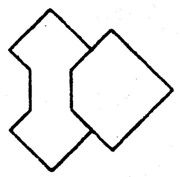
$$I_{total} = I_{int} + I_{ext} + I_{virt} .$$

In figure 16, this summation has been performed for the interpretations of the pattern in figure 3 (the complexities for both interpretations have already been calculated in sections 4, 5, and 6). I_{total} appears to be lower for interpretation B, which was indeed the most preferred interpretation. In order to test I_{total} on a broad range of critical patterns we have made use of published patterns and data stemming from three different papers: Buffart et al (1981)—25 patterns, Boselie (1988)—27 patterns, and Boselie and Wouterlood (1989)—92 patterns. Thus in total 144 patterns have been considered. The first paper was selected because in it the applicability of SIT to completion patterns was claimed. The latter two papers were chosen because they contain a great number of patterns that appear to be in contradiction with predictions made by SIT. In contrast to the present study, in the previous studies predictions were made solely on the basis of the internal structure of interpretations, and an old metric of information, based on memory capacity, was used. The experimental procedure in each of those studies was the same: given a pattern, subjects were asked to draw the contours of their spontaneous pattern interpretation on a piece of paper.

In 52% of all patterns, I_{int} is the lowest for the most preferred interpretation, and therefore correctly predicts that interpretation. In 65% of the patterns, I_{ext} correctly predicts the most preferred interpretation, and in 49% of all patterns this is the case for I_{virt}. However, when taking the sum of these three complexities, it appears that in 95% of the 144 patterns I_{total} correctly predicts the most preferred interpretation. In figure 17 these percentages are depicted. The percentage of correct predictions as a function of the pairwise combinations of the three structure complexities are also depicted.

As can be verified in the histogram in figure 17, there is an overlap in the correct predictions made by the separate structure complexities. The exact overlap is made visible in the Venn diagram shown in figure 18. It can be seen that in 15% of all cases only I_{int} is the lowest for the most-preferred interpretation. In 10% of all cases this holds for I_{ext} and in 14% of all cases for I_{virt}. In 27% of all cases, both I_{int} and I_{ext} correctly predict the most preferred interpretation. In 25% of all cases, this holds for I_{ext} and I_{virt}, and in 7% of all cases for I_{int} and I_{virt}. In only 3% of all patterns are all

Pattern



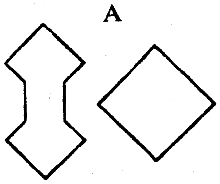
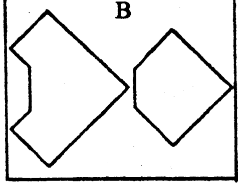
Interpretation	Perceptual complexity			
	I_{int}	I_{ext}	I_{virt}	I_{total}
A 	10	8	1	19
B 	13	0	1	14

Figure 16. The total perceptual complexity of a pattern interpretation is calculated according to the expression: I_{total} — $I_{int} + I_{ext} + I_{virt}$. In this case, it appears that the interpretation with the simplest shapes (A) results in a higher perceptual complexity than does interpretation B, which was the most preferred interpretation.

three complexities the lowest for the most preferred **interpretation.** Notice that for all **examined** patterns at least one of the three complexities is the lowest for the most preferred interpretation. Finally, we examined whether the difference in I_{total} between alternative interpretations for a given pattern is related to the degree of preference for a specific interpretation. This was done by correlating the preference for the simplest-shape interpretation with the proportion I_{total}(simplest-shape interpretation)/I_{total}(second-best interpretation): $r = 0.76$, $p < 0.001$.

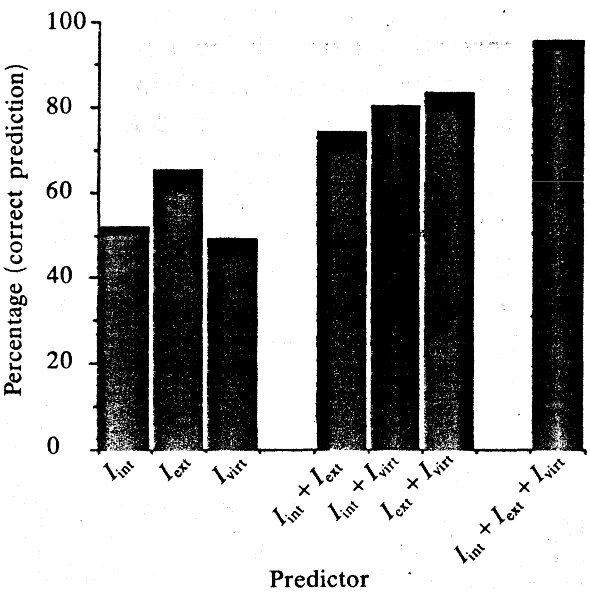


Figure 17. The percentage of patterns for which the most preferred interpretation is predicted correctly according to the three structure complexities and their combinations.

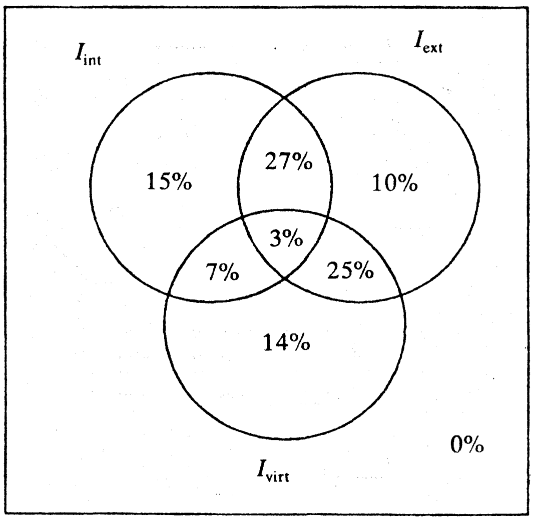


Figure 18. Venn diagram showing the amount of overlap in correct predictions according to the separate structure complexities. Note that for all examined patterns at least one of these complexities is the lowest for the most preferred interpretation.

8 Discussion

For the greater part (95%) of the 144 patterns considered, the most-preferred interpretation is predicted correctly by the global minimum principle, by the use of the proposed quantification of the total perceptual complexity (I_{total}) of interpretations. I_{total} was defined as the sum of the perceptual complexities of the internal structure, the external structure, and the virtual structure of an interpretation. Considering the proportion of correct predictions based on the complexity of each of the three structures separately (52%, 65%, and 49%, respectively), it can be deduced that there is about equal tendency towards interpretations with either the simplest shapes, or the lowest degree of coincidence, or the lowest number of virtual elements. The Venn diagram in figure 18 offers the opportunity to classify patterns according to the predictive quality of those three completion tendencies. In figure 19, a pattern is depicted from every field in the Venn diagram.

Both for the most preferred interpretation and for the second-best interpretation, the complexity of each structure is given. For instance, in figure 19 pattern A, I_{int} alone already predicts the most preferred interpretation correctly, which illustrates the simplest-shape tendency. For other patterns, for instance pattern D in figure 19, introduced by Kanizsa (1985), I_{int} predicts wrongly whereas both I_{ext} and I_{virt} predict correctly. So, for each of the three completion tendencies, there is a restricted class of patterns for which that tendency alone can be considered to be decisive. The three tendencies correspond, to a certain extent, to the principle of pragnanz (taken as focusing on the simplicity of shapes), the coincidence-explanation principle, and the principle of good continuation, respectively. In the past, researchers have sought for support for either this principle or that principle. In the present study, however, we have shown not only that three corresponding tendencies are about equally effective,

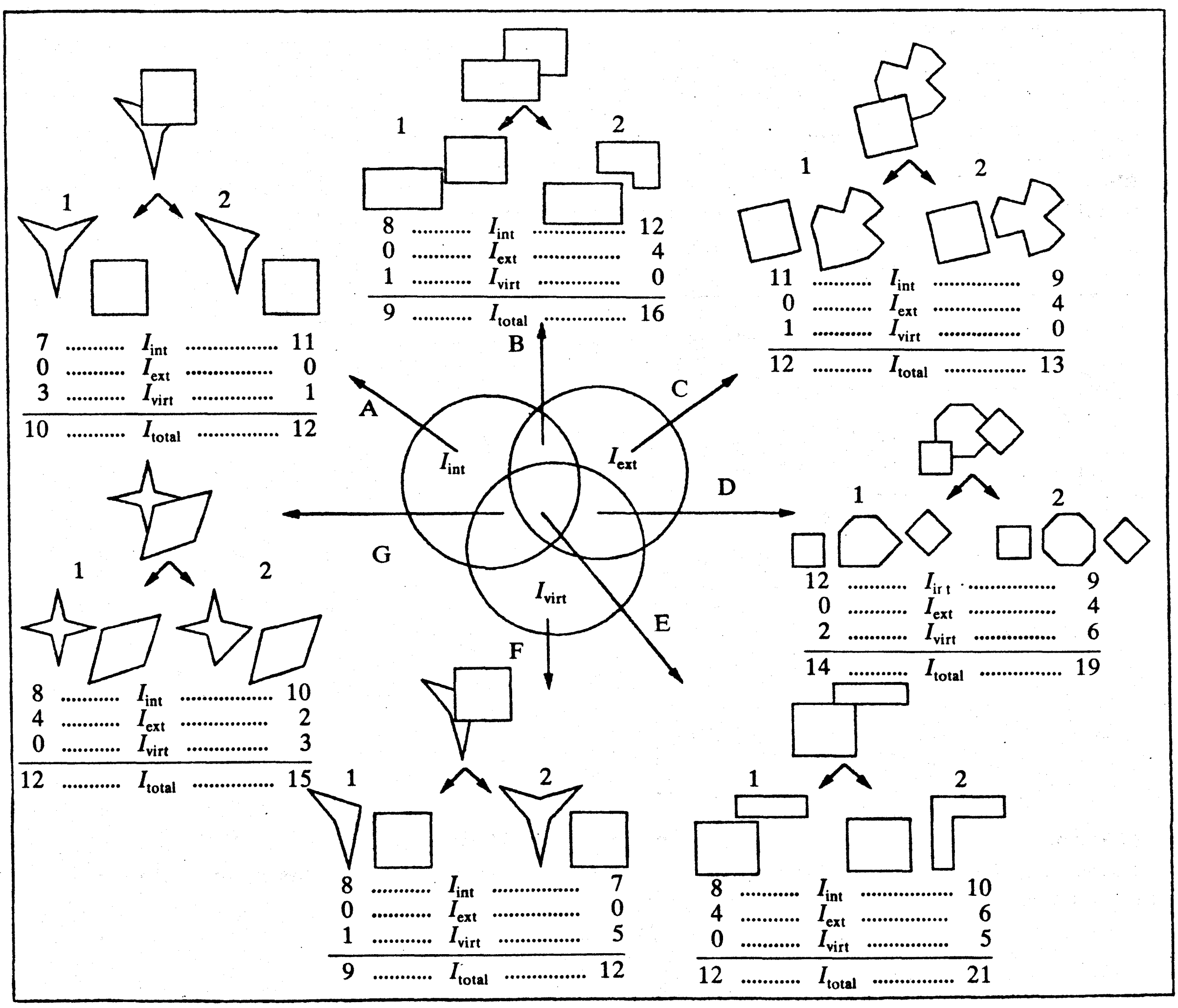


Figure 19. From each area in the Venn diagram shown in figure 18, a pattern is depicted with two possible interpretations. The interpretations labelled with number 1 represent the most preferred interpretation of the pattern. Below each interpretation its complexities are given.

but also that these tendencies can be integrated into one tendency (towards a minimal I_{total}) which is much more effective than each of the three tendencies separately.

Although the proportion of correct predictions on the basis of I_{total} is high, it is expedient to take a closer look at the patterns for which I_{total} leads to a wrong prediction. These wrong predictions seem to be caused by the fact that either I_{ext} or I_{virt} is overestimated. In figure 20a, an example of the first category is given. Although, for the pattern in figure 20a, I_{total} is the same for both interpretations, the mosaic interpretation is highly preferred to the occlusion interpretation. This pattern (figure 20a) is taken from the set of Boselie and Wouterlood (1989). In their study, a mosaic interpretation was assumed whenever subjects did not respond with an occlusion interpretation. We question, however, whether this assumption is justified. Another interpretation is possible and has already been proposed by Kopfermann (1930). In figure 21 a more general pattern of the same type is depicted. This pattern is not easily interpreted as one shape occluding another. A mosaic interpretation with two adjacent shapes is shown as interpretation A. Interpretation B in figure 21 reflects a completely different concept; one where the global contour or envelope plays an important role—a rectangle with a zigzag line in it. In this, the internal structure is determined by the shape of the envelope and the shape of the zigzag. Now, the perceptual complexity of the external structure is determined by the position of the zigzag with respect to the envelope. The perceptual complexities of the external structure of interpretations A and B are 8 and 2, respectively. Indeed, collinearity

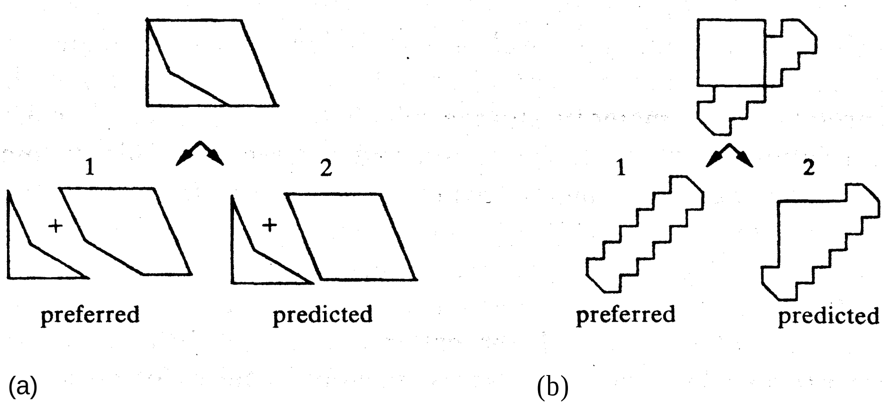


Figure 20, Two patterns, (a) and (b), for which the most preferred interpretation is not predicted correctly by the total perceptual complexity.

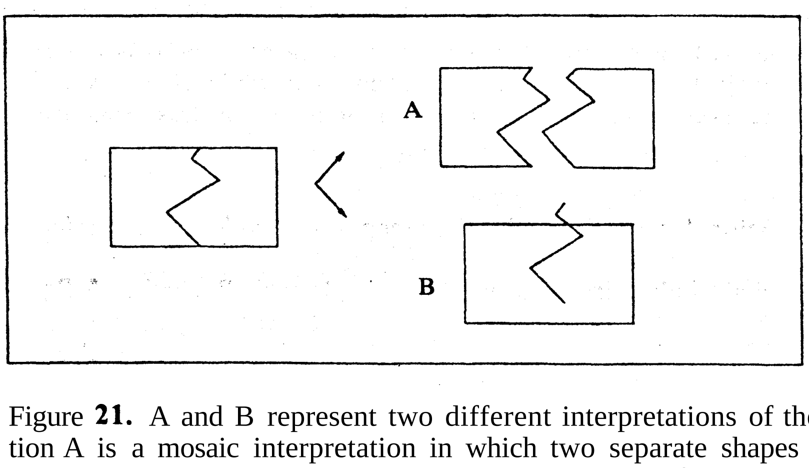


Figure 21. A and B represent two different interpretations of the pattern on the left. Interpretation A is a mosaic interpretation in which two separate shapes are juxtaposed. Interpretation B represents an interpretation in which the envelope of the pattern is taken as one shape. The zigzag as such is taken as a second shape.

enhances the binding between pattern parts and leads, in this case, to an interpretation in which the global contour dominates. This illustrates that envelope interpretations may play a competitive rôle. However, more detailed research has to be focused on this type of interpretation, and we will not consider these interpretations in our analysis.

A pattern belonging to the second category, in which I_{virt} seems to need adjustment, is depicted in figure 20b, stemming from Buffart et al (1981). The total perceptual complexity for the most preferred interpretation, pattern 1, is higher than for interpretation 2, mainly due to the high number of virtual elements in interpretation 1. A reasonable option is that, in addition to the number of virtual elements, the regularity within the virtual structure also plays a role. In order to account for this regularity, one could consider redundancy metrics that are in line with the redundancy concept of Attneave (1954) or with the weight-of-evidence concept of Mackay (1969). Our tentative view, however, is that it is not just the redundancy in the virtual structure that plays a role, but rather an interaction of the redundancies in the virtual structure and in the internal structure. Indeed, the higher the frequency of certain regular substructures in the total pattern, the more likely a completion with this specific regular structure will be made. Any appropriate specification of this type of redundancy requires more research and lies outside the scope of this study.

A further remark on the proposed complexity measure is that the summation of I_{int}, I_{ext}, and I_{virt} is but one of the possible combinations. Other relations between the complexities could be proposed, eventually including weighting factors for a specific type of complexity. Given the present results, however, we conclude that a simple summation already yields a good indicator of the perceptual complexity of an interpretation.

Summarising, the main points in our approach are as follows: (i) A distinction is made between memory complexity and perceptual complexity. That is, it is not the relation between interpretation and memory storage which is relevant, as often has been assumed, but the relation between interpretation and pattern. (ii) This distinction has a great impact on dealing with coincidences caused by the positional relation between the shapes in an interpretation. Although coincidences decrease the memory complexity, they increase the perceptual complexity, not because they are improbable but because they reflect regularities in the external structure of an interpretation. (iii) The perceptual complexity of the internal, the external, and the virtual structure of an interpretation are expressed in the same terms, namely in terms of structural information. In this way, three perceptually relevant tendencies can be combined and subjected to the minimum principle, thereby establishing an integration of global and local aspects of visual occlusion.

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